The associated Sherwood number must also be based on the mean particle diameter:

$$N_{Sh}^* = kd_p/\mathcal{D} \tag{3}$$

The logarithmic regression of N_{Sh}^* against N_{Re}^* for free spheres yields a regression coefficient of 0.77 with a correlation coefficient of 0.916. The theoretical regression coefficient is 0.75 (*9*). This result lends support to the usefulness of Kolmogoroff's (*10*) postulate of the local isotropy of turbulent flow at high Reynolds numbers, as suggested by Middleman (*1*).

Therefore, the mass transfer correlations for the free spheres suggest that the transfer rate is controlled through a turbulent boundary layer, while velocity fluctuations do not disrupt the laminar boundary layer around a fixed particle. The difference between the fixed- and free-sphere experiments also clearly shows that the slip velocity that governs the mass transfer process for freely suspended particles may not be proportional to the mean bulk velocity of the fluid, as Middleman deduces (*1*). It would thus seem inappropriate to use correlations obtained for steady flows over fixed particles as a starting point for describing transport processes in agitated vessels. For the leaching of particles suspended in a turbulent fluid, Levich (*11*) obtains expressions for the mass transfer rate when the particle size is much smaller or much larger than the dissipation scale of turbulence. The predicted dependence of particle diameter on mass transfer coefficient differs from Harriott's experimental relationships (*12*) used by Middleman.

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NOTATION

C_{DF} = skin friction drag coefficient
c_o = concentration at surface, g.-mole/cm.$^{-3}$
$\mathcal{D}$ = diffusivity, sq. cm./sec.$^{-1}$
d_I = diameter of impeller, cm.
d_p = diameter of particle, cm.
d_T = diameter of vessel, cm.
F = Faraday constant, (amp.) (sec.)/g.-mole^{-1}
i_L = limiting current, amp.
k = mass transfer coefficient, cm./sec.$^{-1}$
N = speed of rotation, sec.$^{-1}$
N_{Re} = d_I^2N/ν, impeller Reynolds number
N_{Re}^* = $\rho^{2/3}(P/V)^{1/3}\ d_p^{4/3}/\mu$, isotropic Reynolds number
N_{Sh} = $kd_T/\mathcal{D}$, Sherwood number
N_{Sh}^* = $kd_p/\mathcal{D}$, particle Sherwood number
P = power dissipated, (g.) (cm.)/sec.$^{-1}$
r = radial distance, cm.
S = surface area, sq. cm.
V = volume, cc.
Z = valence change
μ = viscosity, (g.) (cm.$^{-1}$)/sec.$^{-1}$
ν = kinematic viscosity, sq. cm./sec.$^{-1}$
ρ = density, g./cm.$^{-3}$

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Turbulent Motion and Mixing in a Pipe

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Lee and Brodkey (*1*) studied the turbulent mixing of a dye solution injected at the center of a 3-in. pipe. The observed decay of the intensity of concentration fluctuations along the centerline could be approximately predicted using Corrsin's original isotropic turbulent mixer theory (*2*). Since that time Corrsin has modified his theory (*3*). Lee and Brodkey noted in a footnote that preliminary calculations using the revised theory provided a better comparison. This note will present the fully revised computations.

The notation of reference *1* will be used, and equations presented there will not be repeated here. The modification of equations is as follows. Equations (57) and (58) of reference *3* replace Equation (5) of reference *1;* that is, for a low Schmidt number system

$$\tau = \left(\frac{5}{\pi}\right)^{2/3} \frac{2}{(3 - N_{Sc}^2)} \left(\frac{L_s^2}{\epsilon}\right)^{1/3}$$

$$= \left(\frac{\lambda^2}{10\nu}\right) \frac{2}{(3 - N_{Sc}^2)} \left(\frac{k_o}{k_{o,s}}\right)^{2/3} \tag{1}$$

where here and in reference *1,* $2\lambda^2$ is the same as λ_f^2 of reference *3,* and k_o and $k_{o,s}$ are the wave numbers characterizing the energy containing eddies and the principal content of the concentration fluctuations, respectively. For the high Schmidt number system, Equation (70) of reference *3* is the same as Equation (6) of reference *1;* that is

$$\tau = \frac{1}{2}\left[3\left(\frac{5}{\pi}\right)^{2/3} \left(\frac{L_s^2}{\epsilon}\right)^{1/3} + \left(\frac{\nu}{\epsilon}\right)^{1/2} \ln N_{Sc}\right] \tag{2}$$

All the approximations and restrictions suggested in reference *1* still apply and need not be repeated. It would be well to emphasize that the use of these equations does not involve arbitrary constants and that the data to be used for comparison have been adjusted to eliminate the rapid decay that occurs in the wake behind the injector by extrapolating the curve away from the injector back to zero separation and adjusting this to an I_s of unity. The analysis to follow parallels that of reference *1,* so that only points of change will be noted. If it is assumed that $k_o = k_{o,s}$, Equation (1) of this note shows that

$$\left(\frac{5}{\pi}\right)^{2/3} \left(\frac{L_s^2}{\epsilon}\right)^{1/3} = \left(\frac{\lambda^2}{10\nu}\right) \tag{3}$$

This in turn can be used in Equation (2), with ϵ from velocity spectrum measurements, to give the time constant for the decay. Two main assumptions have been made in the analysis. First, $k_o = k_{o,s}$ was assumed valid, since the measurements were made after the dye was dispersed across the pipe cross section. Second, the term of Equation (3) was estimated from the low Schmidt number case, since the scalar macroscale should depend on the manner of injection and not upon the Schmidt number level. The Schmidt number is 7,760, corresponding to a molecular diffusivity of 1.288 × 10^{-6} sq.cm./sec. With this and the other values reported in reference *1,* Equation (2) for the time constant becomes

$$\tau = \frac{1}{2}\,[3(0.544) + (0.01/14.9)^{1/2} \ln 7760]$$

$$= 0.932 \text{ sec.}$$

The earlier calculation based on the theory in reference *2* gave a value of 1.48 sec. In order to compare the calculated decay with the experimental measurements, the time dependency of the decay equation

$$I_s = e^{-t/\tau}$$

is changed to a distance dependency by use of the length traveled by a blob of fluid moving at the local mean velocity, which in this case is the maximum velocity at the centerline; that is

$$t = x/\overline{U}_{\max}$$

The decay is thus expressed as a function of axial distance by multiplying the time constant by $\overline{U}_{\max}$. In terms of feet down the pipe for the present specific case the coefficient for the decay is 0.587 as compared to 0.37 obtained earlier. Figure 1 contains the data of Figure 8 of reference *1* and the new estimations for both the low and high Schmidt number cases. A readjustment of the data has been made due to a small error discovered by Gegner (*4*). This involved a major change for the first three points (about 100%) and a lowering of about 10% for the remaining points. The improvement in the comparison is encouraging and lends support to the analyses of references *1* and *3.* The least squares line through the experimental points gives a slope equivalent to a time constant of 1.03 sec. The old prediction was about 40% high, whereas the new prediction is less than 10% low. The test is quite stringent in that ratios of low to high Schmidt number experiments are not used so that the absolute values of the constants fall out, and there are no arbitrary constants.

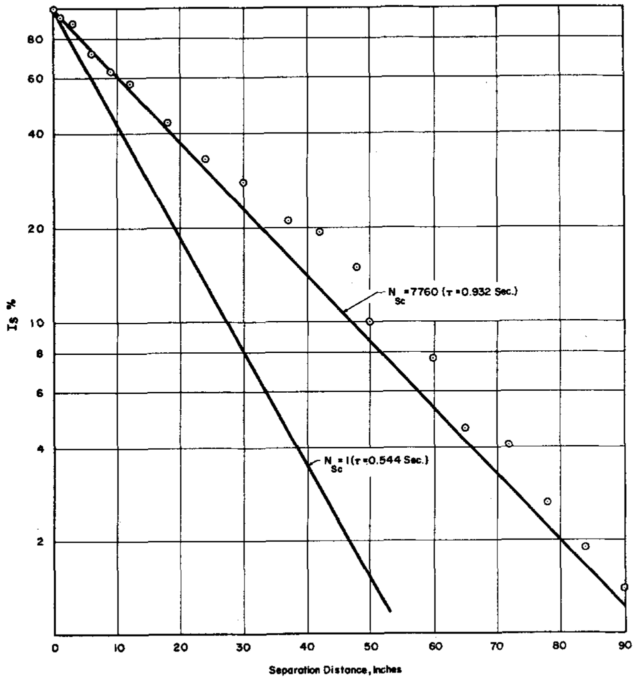

Fig. 1. Intensity of segregation.

As indicated in reference *1,* for general use in mixing calculations the estimation of the microscale from the velocity spectrum is not desirable, since this measurement may not always be available. The only difference in the alternate semiempirical method suggested is in Equation (18) of reference *1* which is replaced by

$$\left(\frac{5}{\pi}\right)^{2/3} \left(\frac{L_s^2}{\epsilon}\right)^{1/3} = \frac{0.341\,R_o}{u_z'} \tag{4}$$

where $k_o = k_{o,s}$ has also been assumed. This estimated would be used in Equation (2) to estimate the time constant. For the one case available, the term estimated from Equation (4) was less than 3% higher than that from Equation (3) using the experimental value of λ.

ACKNOWLEDGMENT

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